

Section 3.4: Polynomial Functions, Graphs

Video 1

1) Sketch the graph of $f(x) = x^3 + 2$, and state the largest open intervals of the domain over which the function is increasing or decreasing.

2) Sketch the graph of $f(x) = (x + 2)^3$, and state the largest open intervals of the domain over which the function is increasing or decreasing.

3) Sketch the graph of $f(x) = -3(x+4)^4 + 1$, and state the largest open intervals of the domain over which the function is increasing or decreasing.

Video 2

Behavior at Zeros

Multiplicity of 1

Odd Multiplicity (> 1)

Even Multiplicity

4) Describe the behavior at each zero: $f(x) = x(x-3)^2(x+2)(x+5)^3(x+8)^6$

Video 3

End Behavior

Odd Degree

Even Degree

5) Use the symbols for end behavior to describe the end behavior of the graph of each function.

a) $f(x) = x^6 + 2x^5 - 17$

b) $f(x) = -x^3 + 8x^2 + 11x + 6$

c) $f(x) = x^5 + 8x^2$

d) $f(x) = -x^4 - 3x^3 + x^2 + 6x - 9$

Video 4

6) Graph the polynomial function $f(x) = 2x^3 + 9x^2 + 7x - 6$.

7) Graph the polynomial function $f(x) = x^4 - 2x^3 - 4x^2 + 2x + 3$.

Video 5

8) Graph the polynomial function $f(x) = (x+1)^3(x-2)^2$.

9) Graph the polynomial function $f(x) = -x(x+4)(x-1)^2$.

Video 6

Intermediate Value Theorem

If $f(x)$ is a polynomial function with real coefficients, and for values a and b we know that $f(a)$ and $f(b)$ have opposite signs, then we know there is at least one real zero between a and b .

10) Use synthetic division to show that $f(x) = x^3 - 2x^2 + 4x - 9$ has at least one real zero between 2 and 3.

Video 7

The Boundedness Theorem

Let $f(x)$ is a polynomial function with real coefficients, degree 1 or higher, and a positive leading coefficient. Suppose $f(x)$ is synthetically divided by $x - c$.

- If $c > 0$ and all of the numbers in the bottom row are nonnegative, then $f(x)$ has no zero greater than c .
- If $c < 0$ and the numbers in the bottom row alternate in sign (0 can be treated as positive or negative), then $f(x)$ has no zero less than c .

11) For $f(x) = x^4 + 5x^2 + 3x - 7$, show that

a) $f(x)$ has no real zero greater than 1

b) $f(x)$ has no real zero less than -2